Space–time control volumes for Caribbean atmospheric reconstruction

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16 October 2026
Research edition 2.0. Working manuscript; not peer reviewed or submitted to arXiv.

Interactive edition: https://climatephysicsai.com/papers/00-space-time.html

Before the paper: the idea in plain language

Imagine an archive in which every small place has a history. A tesseract is our name for one three-dimensional volume followed through time. The research asks whether linking those histories with physical balances helps us reconstruct a poorly observed atmosphere.

How it works, and why it matters

We combine measurements, an imperfect background model and local neural fields. A shared exchange joins neighbouring volumes; uncertainty records the information we still lack.

Value: The aim is a more testable account of moisture transport and local conditions, with a clear path from assumptions to independent checks.

Analogy: A film reel stores successive views of one place. Scrubbing the reel reveals stored frames; recovering a missing frame requires evidence and assumptions.

Abstract

We formulate the dLEGOS tesseract as a spatial control volume crossed with a finite time interval and derive its integrated balance. The representation is connected to hydrostatic mesoscale inference, local neural fields and weak-constraint data assimilation. The central proposed contribution is conditional: whether flux-aware coupling improves recovery of rainfall-relevant atmospheric structure under sparse observations and biased background information. Replay, smoothing, forecasting and scenario response are distinguished. The linked rainfall and temperature studies are empirical component evidence; the full three-dimensional moist reconstruction remains proposed. An interactive Jamaican terrain example makes source support and unresolved fine-scale information visible.

1. Physical question and intended contribution

The organising task is retrospective reconstruction of rainfall-relevant atmospheric structure over Jamaica and, subsequently, other Caribbean islands. The research question is conditional: under which combinations of observation coverage, large-scale driver error and interface formulation does physical coupling improve the recovered moisture transport? Weather prediction, climate response and infrastructure impact are distinct extensions. The use of local networks is a representation choice, not evidence that a new physical inference principle has been discovered.

The latest doctoral review places this question beside finite-basis PINNs, conservative domain decomposition and weak-constraint data assimilation. These are the nearest methodological lineages. A useful contribution must demonstrate a change in withheld physical information or computational cost after matching the available observations, model capacity and temporal window. [1] [2] [3]

2. A space–time cell and the tesseract analogy

Let Ω_i be a spatial control volume and $I_n=[t_n,t_{n+1}]$ a time interval. The computational block is $K_i^n=\Omega_i\times I_n$. When Ω_i is a cube, this product has the topology of a four-dimensional hypercube: 16 vertices, 32 edges, 24 square faces and eight cubic facets. Its two temporal facets are the spatial volume at the beginning and end of the interval; six remaining facets are the spatial faces swept through time. A drawing is necessarily a projection. Atmospheric variables and physical time remain dimensioned quantities; visual rotation between axes is not a physical symmetry.

$$\mathcal{K}_i^n = \Omega_i \times [t_n, t_{n+1}], \qquad \xi = \left(\frac{x - x_i}{L_x}, \frac{y - y_i}{L_y}, \frac{z - z_i}{L_z}, \frac{t - t_n}{\tau} \right). \tag{1}$$

Normalised coordinates are dimensionless. Lx, Ly, Lz and τ are prescribed scales; using one drawing scale does not equate seconds with metres.

Interactive component in the online edition: https://climatephysicsai.com/papers/

An original archive instrument built with Anatomy's isometric SVG kit. Each tray holds a cubic spatial sample; the illuminated tray selects a stored time. This is a conceptual instrument, not a physical atmospheric apparatus.

> **Observation**A hypercube is not an additional atmospheric force. Its value is bookkeeping: a state, its interfaces, its observation window and its provenance can be inspected together.

3. Derivation of the space–time balance

Take an extensive density u, a spatial flux F and source s satisfying $\partial_t u+\nabla\cdot F=s$ in a fixed spatial cell. Integrate first over Ω_i and then over I_n. The time derivative

supplies the difference between the temporal facets, while the spatial divergence supplies the outward transfer through the lateral facets. This is the operational mathematical meaning of the space–time block.

$$\int_{\Omega_i} u(\mathbf{x}, t_{n+1})\, dV - \int_{\Omega_i} u(\mathbf{x}, t_n)\, dV + \sum_{f \subset \partial\Omega_i} \int_{t_n}^{t_{n+1}} \int_f \mathbf{F} \cdot \mathbf{n}_{if}\, dA\, dt = \int_{t_n}^{t_{n+1}} \int_{\Omega_i} s\, dV\, dt.$$

(2)

u has the units of the conserved quantity per m^3; F has those units per m^2 per s; s has those units per m^3 per s. Every term is an extensive amount.

Define U_i^n as the cell integral, $\Phi_i f^n$ as the integrated outward face transfer and S_i^n as the integrated source. A shared face between i and j has $\Phi_i f^n = -\Phi_j f^n$ when the same transfer is assigned with opposite orientations. Summation removes internal faces and leaves temporal storage, outer-boundary exchange and sources. This cancellation does not assert that a learned flux equals the true atmospheric flux.

$$U_i^{n+1} = U_i^n - \sum_f \Phi_{if}^n + S_i^n, \qquad \sum_i (U_i^{n+1} - U_i^n) = -\sum_{f \in \partial\Omega} \Phi_f^n + \sum_i S_i^n.$$

(3)

Fixed geometry is assumed. Moving boundaries require the corresponding relative flux; terrain-cut volumes require correct measures.

For total atmospheric water, the state must include the selected water species and all transfers represented by the closure. A water-vapour equation alone does not supply rain microphysics. For a vertically integrated total-water budget, $dW/dt + \nabla_h \cdot IVT = E - P$ requires explicit conventions for condensate storage, top exchange and the lower boundary. An analysis increment belongs in the source ledger. Paper 1 develops a deliberately smaller sensible-heat system before the moist extension. [4]

4. Restricted atmospheric state and neural parameterisation

The initial real-atmosphere inference proposal uses hydrostatic mesoscale fields on pressure surfaces, consistent with the revised literature review. Let h=(u,v) be horizontal velocity, T temperature, q water-vapour specific humidity, Φ geopotential and ω=Dp/Dt pressure velocity. Surface pressure, terrain masks, lateral background information and uncertain prescribed tendencies are retained. The hydrostatic class does not resolve individual nonhydrostatic convective updraughts merely because a fine display grid is chosen.

$$\frac{D\mathbf{h}}{Dt} + f\mathbf{k} \times \mathbf{h} = -\nabla_p \Phi + \mathbf{F}_h, \quad \frac{\partial \Phi}{\partial p} = -\frac{R_d T_v}{p}, \quad \nabla_p \cdot \mathbf{h} + \frac{\partial \omega}{\partial p} = 0,$$

(4)

D/Dt=∂t+h·∇p+ω∂p. u,v: m s^-1; ω: Pa s^-1; Φ: m² s^-2; f: s^-1; Rd: J kg^-1 K^-1; Tv is virtual temperature.

$$\frac{DT}{Dt} - \frac{\kappa T \omega}{p} = \frac{Q}{c_p}, \qquad \frac{Dq}{Dt} = S_q, \qquad \kappa = R_d/c_p.$$

(5)

Q: W kg^-1, Sq: kg kg^-1 s^-1. Momentum tendency Fh, heating and moisture tendency require declared closures or bounded discrepancy priors.

A local neural field fθᵢ maps normalised space–time coordinates and admissible context to resolved variables. Differentiation supplies PDE residuals. Geopotential can be diagnosed by hydrostatic integration with a stated boundary value; pressure velocity requires compatible continuity and mass boundaries. Allowing arbitrary unknown forcing to absorb every residual would make a good fit uninformative. The first observing-system experiment therefore fixes or bounds those terms. Paper 4 specifies the local network, shared flux and optimisation alternatives. [5]

$$\hat{\mathbf{x}}(\xi) = \sum_i \chi_i(\xi)\, f_{\theta_i}(\xi, \mathbf{c}_i), \quad \sum_i \chi_i = 1, \qquad \chi_i \geq 0.$$

(6)

A partition of unity blends overlapping local fields. Smooth blending alone does not guarantee conservative fluxes; the shared transfer must be defined separately.

5. Reconstruction, forecasting and the direction of time

A replay reads a stored state at a chosen timestamp. A reconstruction estimates a state from incomplete observations and a physical prior. A filter conditions only on observations available up to the estimated time; a smoother conditions on a declared complete window. An operational forecast must exclude later observations and later product revisions. These differences concern information, not animation direction.

$$\text{filter: } p(\mathbf{x}_t \mid \mathbf{y}_{0:t}), \quad \text{smoother: } p(\mathbf{x}_t \mid \mathbf{y}_{0:T}), \quad \text{forecast: } p(\mathbf{x}_{t+h} \mid \mathbf{y}_{0:t}),\ T > t.$$

(7)

Later information can support an improved historical estimate; it cannot be credited to an earlier forecast.

One reference objective is weak-constraint four-dimensional variational estimation. A neural field changes its parameterisation, while a discrete or continuous residual changes the evolution discrepancy. It does not remove the inverse problem or establish identifiability. [2]

$$J = \tfrac{1}{2}\|\mathbf{x}_0 - \mathbf{x}_b\|^2_{B^{-1}} + \tfrac{1}{2}\sum_n \|H_n(\mathbf{x}_n) - \mathbf{y}_n\|^2_{R_n^{-1}} + \tfrac{1}{2}\sum_n \|\mathbf{x}_{n+1} - M_n(\mathbf{x}_n)\|^2_{Q_n^{-1}}.$$

(8)

B, R and Q describe background, observation and model-discrepancy covariance; H maps model states to the actual measurement support. Temporally correlated errors require joint covariance.

Dissipation is a concrete obstacle to a literal time reversal. A Fourier mode of the heat equation decays as $\exp(-\kappa k^2 t)$. Backward inversion multiplies error by $\exp(\kappa k^2 t)$, amplifying the smallest-scale noise fastest. A visually reversible slider therefore re-indexes stored observations; it does not solve a stable inverse heat equation. Conditional historical inference must report uncertainty and prior sensitivity.

6. Falsifiable hypotheses and required evidence

Hypothesis	Controlled comparison	Outcome that could reject it
H1: physical aggregation helps under sufficiently accurate forcing	Hard, soft, unconstrained and seasonal allocation; controlled driver biases	Independent error grows while aggregate error is zero
H2: flux coupling improves moisture reconstruction under sparsity	Uncoupled, state-coupled, flux-coupled, global neural field and assimilation	No improvement in withheld transport at matched data and compute
H3: explicit discrepancy improves probabilistic reconstruction	Matched uncertainty models with/without background and closure uncertainty	Poorer proper score or dependence despite wider intervals

Begin with a known-state observing-system experiment, then vary station geometry, vertical information, noise and boundary bias. The real-event stage should use withheld observations and independently assembled budget terms where available. Measured throughput, peak memory, training cost, inference cost and error-versus-cost curves are required before claiming computational efficiency. A smaller network is not automatically a cheaper end-to-end inference procedure.

> **Observation**The actual rainfall and temperature component studies constrain expectations. They show that exact accounting or a physics loss can help a matched model while a simpler or frozen comparator remains stronger. Papers 3 and 5 report those outcomes.

7. Motivation, regional value and limits

My interest in the tesseract was inspired by Stargate SG-1 and Star Trek: fictional journeys through unfamiliar dimensions made the idea of examining one place across time compelling. Here that inspiration is a visual analogy. The scientific object is an ordinary space–time control volume, and returning to the past means testing a reconstruction against evidence. The attraction is being able to ask what happened within a subregion, which clues constrain that account, and what remains unknown.

For Jamaica, a physically interpretable reconstruction could help examine terrain–coast contrasts, moisture inflow and the environmental conditions surrounding a historical event. Globally, the intended contribution is a reproducible account of the regimes in which local coupling improves inference or transmits driver error. Infrastructure planning is a later decision application: it needs building geometry, surface properties, impact-specific physics and external validation. Paper 7 makes that extension explicit rather than treating a rendered city as an established prediction.

> **Literature update · 7 October 2026**The distinction between weather prediction and response to altered forcing is also emphasised in the recent Beucler et al. preprint. Climate claims require explicit forcing and stress tests outside the historical distribution. [6] Compare the closest literature.

References and source lineage

Citations resolve to the shared numbered bibliography. The source-access register retains inherited and unappraised records separately. [5] [2] [1] [7]

1. [1] Dolean, V.; Heinlein, A.; Mishra, S.; Moseley, B. (2022). Finite basis physics-informed neural networks as a Schwarz domain decomposition method. arXiv:2211.05560v2 [preprint]. Included in the revised doctoral literature review; full-text and identity checks vary by source
2. [2] Trémolet, Y. (2006). Accounting for an imperfect model in 4D-Var. Quarterly Journal of the Royal Meteorological Society, 132(621), 2483-2504. Included in the revised doctoral literature review; full-text and identity checks vary by source
3. [3] Bocquet, M.; Sakov, P. (2013). Joint state and parameter estimation with an iterative ensemble Kalman smoother. Nonlinear Processes in Geophysics, 20(5), 803-818. Included in the revised doctoral literature review; full-text and identity checks vary by source
4. [4] Skamarock et al. (2019). A Description of the Advanced Research WRF Model Version 4. NCAR Technical Note NCAR/TN-556+STR.
5. [5] Raissi, Perdikaris & Karniadakis (2019). Physics-informed neural networks: A deep learning framework for solving forward and inverse problems

involving nonlinear partial differential equations. Journal of Computational Physics, 378, 686–707.

6. [6] Beucler, T. et al. (2026). Artificial intelligence pathways from weather to climate. arXiv:2610.09770v1. Preprint submitted 7 October 2026. Primary abstract and relevant full-text sections inspected 16 October 2026; preprint, not journal acceptance

7. [7] Harder, P.; Hernandez-Garcia, A.; Ramesh, V.; Yang, Q.; Sattegeri, P.; Szwarcman, D.; Watson, C.; Rolnick, D. (2023). Hard-Constrained Deep Learning for Climate Downscaling. Journal of Machine Learning Research, 24(365), 1–40. Included in the revised doctoral literature review; full-text and identity checks vary by source

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